Activity Plan Template

Title: Where Should You Build? Investigating Probability and Expected Value in Catan

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Topic: Theoretical probability, experimental probability, and expected resource production with two fair dice.

Connection to Core Curriculum: Utah Secondary Mathematics II, S.CP.1 (Statistics and Probability): Describe events as subsets of a sample space using characteristics of outcomes, including unions ("or"). Students identify the sums that trigger each settlement and combine mutually exclusive dice-sum events to find their probabilities. The activity also develops interpretation of simulation-based relative frequencies.

Overview: Participants use the two-page Catan Settlement Challenge worksheet to compare hypothetical Settlement A (4, 5, 8) and Settlement B (9, 10, 12). They predict a location, use the 6 × 6 two-dice sum table to calculate theoretical probabilities, and use AZRS Probability Lab to compare experimental resource production after 10, 110, and (if time allows) 1,110 cumulative rolls. Each matching roll earns one resource; dice are fair and no robber blocks production. The worksheet's board illustration provides context, while the listed number sets define the investigation.

Objectives: By the end, participants will (1) count favorable outcomes from the 36 equally likely ordered two-dice outcomes; (2) compute P(A) = (3+4+5)/36 = 12/36 = 33.33% and P(B) = (4+3+1)/36 = 8/36 = 22.22%; (3) calculate experimental production rates from simulated frequency counts; and (4) explain the difference among theoretical probability, expected resource count, experimental rate, and a guaranteed outcome.

Materials Needed: Two-page Catan Settlement Challenge worksheet (board illustration, two-dice sum table, experiment recording table, reasoning questions); projector; internet-connected instructor computer; optional student devices; pencils and calculators; AZRS Probability Lab link; instructor backup screenshots or counts for 10, 110, and 1,110 rolls.

Technology: AZRS Probability Lab (Coins & Dice), "Two dice (sum)" mode, accessed in a web browser; classroom projection for displaying the simulation and frequency graph.

Role of Technology: AZRS Probability Lab simulates many rolls of two fair dice and displays the observed frequency for each sum alongside theoretical probability markers. Participants add frequencies for A (4, 5, 8) and B (9, 10, 12), compute production rates, and compare short-run variation with long-run patterns. The tracked-outcome display highlights only one sum, so the full histogram must be used to calculate settlement totals. The applet enables a 1,110-roll comparison that would be impractical with physical dice in a 10-minute lesson.

Web Reference: https://www.azrscorp.com/quickfix/simulations/probability-lab/ (select "Two dice (sum)"). Add this hyperlink to the course webpage before class.

Activity Plan:

1. Launch and predict. Distribute the two-page worksheet. Show the Catan board illustration and explain the simplified rule: a settlement receives one resource when the sum of two dice matches one of its three listed numbers. State the comparison explicitly: A = (4, 5, 8), B = (9, 10, 12). Ask participants to choose a settlement before calculating and justify their prediction to a neighbor. The illustration is contextual; use the stated number sets for all calculations.

2. Establish theoretical expectations. Have pairs use the worksheet's 6 × 6 sum table to count ways to obtain 4 (3), 5 (4), 8 (5), 9 (4), 10 (3), and 12 (1). Since only one sum occurs per roll, add favorable counts: A = 12/36 = 33.33%; B = 8/36 = 22.22%. Expected resources over 10 rolls are 10(12/36) = 3.33 for A and 10(8/36) = 2.22 for B. Ask why expected counts need not be whole numbers and whether a prediction guarantees a result.

3. Run the short simulation. Open the linked AZRS Probability Lab and select "Two dice (sum)." Reset and simulate 10 rolls. Participants record frequencies for sums 4, 5, 8, 9, 10, and 12 in the worksheet, add A and B totals, and divide each total by 10. Ask whether the observed production matches the expectation. Instructor backup example from a prior run: A = 6/10 = 60%, B = 1/10 = 10%; use live results if available.

4. Extend the evidence. Without resetting, add 100 rolls for 110 cumulative trials. Participants record the six frequencies, calculate each total divided by 110, and compare with 33.33% and 22.22%. If time allows, add 1,000 more rolls for 1,110 cumulative trials; otherwise display the backup screenshot and discuss the worksheet's 1,110-roll row. Prior-run backup counts: after 110 rolls, A = 43/110 = 39.09%, B = 20/110 = 18.18%; after 1,110 rolls, A = 375/1110 = 33.78%, B = 234/1110 = 21.08%. Explain that convergence need not improve after every batch.

5. Discuss and assess. Use worksheet Questions 8–11 as brief oral prompts: How do experimental and theoretical rates compare? Does higher expected production guarantee more resources in every game? Could B outperform A in 10 rolls? If A's theoretical rate is 33.33% but it produces six resources in 10 rolls, is the theory wrong? Expected reasoning: short runs vary; expected values are averages, not guarantees; long-run relative frequencies tend toward theoretical rates. Exit ticket ("Your strategy"): choose A or B, cite one calculation and one experimental observation, and explain what the choice does not guarantee. Participants may complete longer written responses after class.

Background: In Catan, a roll of two dice determines which numbered terrain hexes produce resources for adjacent settlements. The numbers are not equally likely: 7 has six ordered combinations, whereas 2 and 12 have one each. This simplified investigation isolates probability from other strategic considerations, such as resource diversity, ports, the robber, and competing players.

Extensions: In a longer lesson, students can run independent 1,110-roll trials and compare rates across groups; calculate expected resource counts for 100 or 1,000 rolls; examine how a city doubles resource output; or compare probabilities with resource diversity, ports, and trading. Ask whether the experimental rate must get closer to theory after every added batch (no).

Included documents: Final two-page "CATAN SETTLEMENT CHALLENGE" student worksheet. Page 1 includes the game context, settlement choices, and 6 × 6 two-dice sum table. Page 2 includes theoretical probability questions, a frequency table for 10, 110, and 1,110 rolls, experimental rate calculations, reasoning questions 8–11, and a final strategy statement. Instructor backup results are described in Steps 3–4.

References: AZRS Corporation. Probability Lab (Coins & Dice). https://www.azrscorp.com/quickfix/simulations/probability-lab/ ; CATAN GmbH. CATAN game rules. https://www.catan.com/understand-catan/game-rules ; Utah State Board of Education. Utah Core Standards for Secondary Mathematics (S.CP.1).