**The Number ZERO**

By Lane Messick

Zero. Where did the number zero come from? How long has zero been around? Who came up with the symbol zero? Next time you are driving through the country look and try to count the number of cows. I am sure that you will not start with the number zero. You will most likely start with one. Why is that? Don't we want to know that you are starting to count by starting with zero? I mean when you don't see any cows you have counted zero cows. Now let’s continue to count, one, two, three, ..., nine, ten. Now what is the number ten? You have a one in the ones place but you put a zero in for the tens place. Why is that? My guess is that you need to show that you have nothing in the tens place. It actually is not a guess but you would need to have some way of noting that there is nothing in the tens place. The reason is because if you are trying to tell the difference from two thousand five hundred and seven (2507) from two hundred and fifty seven (257). Without that zero in the tens place the number changes completely.

You also have the problem of knowing exactly what one means. If I say “three fifty”. What is the first thing that comes to your mind? 350 Miles? $350? Or even $3.50? We have a way of interpreting that but we all use slang to interpret numbers. That is the same as in ancient times. There times where the number were used in slang but the zero made since. In the history section you will be able to get a better explanation of how that is and why.

**History and Background**

One might think that once a place-value number system came into existence then the 0 as an empty place indicator is a necessary idea, yet the Babylonians had a place-value number system without this feature for over 1000 years. Moreover there is absolutely no evidence that the Babylonians felt that there was any problem with the ambiguity which existed. Remarkably, original texts survive from the era of Babylonian mathematics. The Babylonians wrote on tablets of unbaked clay, using cuneiform writing. The symbols were pressed into soft clay tablets with the slanted edge of a stylus and so had a wedge-shaped appearance. Many tablets from around 1700 BC survive and we can read the original texts. Of course their notation for numbers were quite different from ours. Not only was it different the Babylonians used a base 60 instead of the base 10 that we use today. To translate into our notation they would not distinguish between 2507 and 257. It was not until around 400 BC that the Babylonians put two wedge symbols into the place where we would put zero to indicate which was meant, 257 or 25 '' 7.

The two wedges were not the only notation used, however, and on a tablet found at Kish, an ancient Mesopotamian city located east of Babylon in what is today south-central Iraq, a different notation is used. This tablet, thought to date from around 700 BC, uses three hooks to denote an empty place in the positional notation. Other tablets dated from around the same time use a single hook for an empty place. There is one common feature to this use of different marks to denote an empty position. This is the fact that it never occurred at the end of the digits but always between two digits. So although we find 25 '' 7 we never find 257 ''. One has to assume that the older feeling that the context was sufficient to indicate which was intended still applied in these cases.

Now the ancient Greeks began their contributions to mathematics around the time that zero as an empty place indicator was coming into use in Babylonian mathematics. The Greeks however did not adopt a positional number system. It is worth thinking just how significant this fact is. How could the brilliant mathematical advances of the Greeks not see them adopt a number system with all the advantages that the Babylonian place-value system possessed? The real answer to this question is more subtle than the simple answer that we are about to give, but basically the Greek mathematical achievements were based on geometry. Although [Euclid](http://www-history.mcs.st-and.ac.uk/Mathematicians/Euclid.html)'s Elements contains a book on number theory, it is based on geometry. In other words Greek mathematicians did not need to name their numbers since they worked with numbers as lengths of lines. Numbers which required to be named for records were used by merchants, not mathematicians, and hence no clever notation was needed.

Now there were exceptions to what we have just stated. The exceptions were the mathematicians who were involved in recording astronomical data. Here we find the first use of the symbol which we recognize today as the notation for zero, for Greek astronomers began to use the symbol O. There are many theories why this particular notation was used. Some historians favor the explanation that it is omicron, the first letter of the Greek word for nothing namely "ouden". [Neugebauer](http://www-history.mcs.st-and.ac.uk/Mathematicians/Neugebauer.html), however, dismisses this explanation since the Greeks already used omicron as a number - it represented 70 (the Greek number system was based on their alphabet). Other explanations offered include the fact that it stands for "obol", a coin of almost no value, and that it arises when counters were used for counting on a sand board. The suggestion here is that when a counter was removed to leave an empty column it left a depression in the sand which looked like O.

[Ptolemy](http://www-history.mcs.st-and.ac.uk/Mathematicians/Ptolemy.html) in the Almagest written around 130 AD uses the Babylonian sexagesimal system together with the empty place holder O. By this time [Ptolemy](http://www-history.mcs.st-and.ac.uk/Mathematicians/Ptolemy.html) is using the symbol both between digits and at the end of a number and one might be tempted to believe that at least zero as an empty place holder had firmly arrived. This, however, is far from what happened. Only a few exceptional astronomers used the notation and it would fall out of use several more times before finally establishing itself. The idea of the zero place makes its next appearance in Indian mathematics.

The scene now moves to India where it is fair to say the numerals and number system was born which have evolved into the highly sophisticated ones we use today. Of course that is not to say that the Indian system did not owe something to earlier systems and many historians of mathematics believe that the Indian use of zero evolved from its use by Greek astronomers. As well as some historians who seem to want to play down the contribution of the Indians in a most unreasonable way, there are also those who make claims about the Indian invention of zero which seem to go far too far.

What is certain is that by around 650AD the use of zero as a number came into Indian mathematics. The Indians also used a place-value system and zero was used to denote an empty place. In fact there is evidence of an empty place holder in positional numbers from as early as 200AD in India but some historians dismiss these as later forgeries. Let us examine this latter use first since it continues the development described above.

In around 500AD [Aryabhata](http://www-history.mcs.st-and.ac.uk/Mathematicians/Aryabhata_I.html) devised a number system which has no zero yet was a positional system. He used the word "kha" for position and it would be used later as the name for zero. There is evidence that a dot had been used in earlier Indian manuscripts to denote an empty place in positional notation. It is interesting that the same documents sometimes also used a dot to denote an unknown where we might use x. Later Indian mathematicians had names for zero in positional numbers yet had no symbol for it. The first record of the Indian use of zero which is dated and agreed by all to be genuine was written in 876.

We have an inscription on a stone tablet which contains a date which translates to 876. The inscription concerns the town of Gwalior, 400 km south of Delhi, where they planted a garden 187 by 270 hastas which would produce enough flowers to allow 50 garlands per day to be given to the local temple. Both of the numbers 270 and 50 are denoted almost as they appear today although the 0 is smaller and slightly raised.

We now come to considering the first appearance of zero as a number. Let us first note that it is not in any sense a natural candidate for a number. From early times numbers are words which refer to collections of objects. Certainly the idea of number became more and more abstract and this abstraction then makes possible the consideration of zero and negative numbers which do not arise as properties of collections of objects. Of course the problem which arises when one tries to consider zero and negatives as numbers is how they interact in regard to the operations of arithmetic, addition, subtraction, multiplication and division.

Perhaps we should note at this point that there was another civilization which developed a place-value number system with a zero. This was the Maya people who lived in Central America, occupying the area which today is southern Mexico, Guatemala, and northern Belize. This was an old civilization but flourished particularly between 250 and 900. We know that by 665 they used a place-value number system to base 20 with a symbol for zero. However their use of zero goes back further than this and was in use before they introduced the place-valued number system. This is a remarkable achievement but sadly did not influence other people.

The brilliant work of the Indian mathematicians was transmitted to the Islamic and Arabic mathematicians further west. It came at an early stage for [al-Khwarizmi](http://www-history.mcs.st-and.ac.uk/Mathematicians/Al-Khwarizmi.html) wrote Al'Khwarizmi on the Hindu Art of Reckoning which describes the Indian place-value system of numerals based on 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0. This work was the first in what is now Iraq to use zero as a place holder in positional base notation. Ibn [Ezra](http://www-history.mcs.st-and.ac.uk/Mathematicians/Ezra.html), in the 12th century, wrote three treatises on numbers which helped to bring the Indian symbols and ideas of decimal fractions to the attention of some of the learned people in Europe.

The Indian ideas spread east to China as well as west to the Islamic countries. In 1247 the Chinese mathematician [Ch'in](http://www-history.mcs.st-and.ac.uk/Mathematicians/Ch%27in.html) Chiu-Shao wrote Mathematical treatise in nine sections which uses the symbol O for zero. A little later, in 1303, [Zhu Shijie](http://www-history.mcs.st-and.ac.uk/Mathematicians/Zhu_Shijie.html) wrote Jade mirror of the four elements which again uses the symbol O for zero.

[Fibonacci](http://www-history.mcs.st-and.ac.uk/Mathematicians/Fibonacci.html) was one of the main people to bring these new ideas about the number system to Europe. Fibonacci learned them from Arab traders, who presumably adopted them during travels to the Indian subcontinent

In Liber Abaci he described the nine Indian symbols together with the sign 0 for Europeans in around 1200 but it was not widely used for a long time after that. It is significant that [Fibonacci](http://www-history.mcs.st-and.ac.uk/Mathematicians/Fibonacci.html) is not bold enough to treat 0 in the same way as the other numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 since he speaks of the "sign" zero while the other symbols he speaks of as numbers.

One might have thought that the progress of the number systems in general, and zero in particular, would have been steady from this time on. However, this was far from the case. [Cardan](http://www-history.mcs.st-and.ac.uk/Mathematicians/Cardan.html) solved cubic and quartic equations without using zero. He would have found his work in the 1500's so much easier if he had had a zero but it was not part of his mathematics. By the 1600's zero began to come into widespread use but still only after encountering a lot of resistance.

Of course there are still signs of the problems caused by zero. Not to long agou many people throughout the world celebrated the new millennium on 1 January 2000. Of course they celebrated the passing of only 1999 years since when the calendar was set up on year zero was specified. Although one might forgive the original error, it is a little surprising that most people seemed unable to understand why the third millennium and the 21st century begin on 1 January 2001. Zero is still causing problems!

**Explanations of mathematics**

Charles Seif's Proof from his Book: a Biography of a Dangerous Idea

Let a = 1 and b = 1 such that b2 = ab = 1.
Now let's start out with a simple equation.
a2 = a2
We will now subtract one from both sides and we will now have;
a2 - 1 = a2 - 1
Now let's substitute what was given to us, let one of the 1's be ab and the other 1 be b^2. We now have:
a2 – b2 = a2 - ab
as you can see we have not changed the number at all. Now let's simplify.
a2 – b2 simplified = (a - b)(a + b)
a2 - ab simplified = a(a - b)
so we now have
(a - b)(a + b) = a(a - b)
Now we can see that we have like terms and so we will divide by (a - b) from both sides:
We have:
a + b = a
Now subtract a from both sides we get
b = 0
Now b was given to us so we now have
1 = 0
This is very SCARY. What went wrong? We can never have 1 = 0. If we go back up to when we divided (a - b) we are essentially dividing (1 - 1) which is zero and we cannot divide by zero.

**Significance and applications**

On my website I posted a video and this is the dialog that I put on my video which helps ups understand the Multiplication Property of Zero

Today I am going to be talking about the Multiplication Property of zero. We all have friends. Some of them are a little corky. You know they just do things like no one else. We all have them. This is a good thing. Because of them the world is a better place and that is because we are not all the same. Usually those people that are the most different, make life the most interesting.

In the Math world zero is that corky friend. It does not follow the rules of all the other numbers. It has its own unique properties that makes mathematics more interesting. One of zero’s unique rules is called the multiplication property of Zero. The multiplication property states that: The product of 0 and any other number results in 0. That is for any real number a, a times 0 = 0. It does not matter what the number is and when you multiply it to zero, you get zero as the answer. That means:
3 X 0 = 0           6 X 0 = 0
0 X -5 = 0           -8 X 0 = 0
It is like saying that you have 3 groups of zero is zero, or even, zero groups of -5 is zero.
It is the same for any algebraic terms:
b X 0 = 0           y^2 X 0 = 0
And even x^2y^3z^15 = 0
It does not matter what else is going on because when you have a number times zero you will always always get zero. That is the Multiplication Property of zero.

**References**

**Books:**

1. R Calinger, *A conceptual history of mathematics* (Upper Straddle River, N. J., 1999).
2. G Ifrah, *From one to zero : A universal history of numbers* (New York, 1987).
3. G Ifrah, *A universal history of numbers : From prehistory to the invention of the computer* (London, 1998).
4. G G Joseph, *The crest of the peacock* (London, 1991).
5. R Kaplan, *The nothing that is : a natural history of zero* (London, 1999).
6. R Mukherjee, *Discovery of zero and its impact on Indian mathematics* (Calcutta, 1991).

**Articles:**

1. S Giuntini, A discussion concerning the nature of zero and the relation between imaginary and real numbers (Italian), *Boll. Storia Sci. Mat.* **4** (1) (1984), 25-63.
2. R C Gupta, Who invented the zero?, *Ganita-Bharati* **17** (1-4) (1995), 45-61.
3. P Mäder, "Wie die Puppe ein Adler sein wollte, der Esel ein Löwe, die Äffin eine Königin - so wollte die Null eine Ziffer sein!" Ein Überblick zur Geschichte der Zahl Null, in *Jahrbuch Überblicke Mathematik, 1995* (Braunschweig, 1995), 39-64.
4. R N Mukherjee, Background to the discovery of the symbol for zero, in Proceedings of the Symposium on the 1500th Birth Anniversary of Aryabhata I, New Delhi, 1976, *Indian J. Hist. Sci.* **12** (2) (1977), 225-231.
5. K Muroi, The expressions of zero and of squaring in the Babylonian mathematical text VAT 7537, *Historia Sci.* (2) **1** (1) (1991), 59-62.
6. L Pogliani, M Randic and N Trinajstic, Much ado about nothing - an introductive inquiry about zero, *Internat. J. Math. Ed. Sci. Tech.* **29** (5) (1998),729--744.
7. S Ursini Legovich, The origin of the zero in Central American civilization. Comparative analysis with the Hindu case (Spanish), *Mat. Enseñanza No.* **13** (1980), 7-20.
8. M Ja Vygodskii, L'origine du signe de zéro dans la numération babylonienne (Russian), *Istor.-Mat. Issled.* **12** (1959), 393-420.

**Websites:**

1. <https://www.quora.com/Who-invented-zero-and-how>
2. <http://www.und.edu/instruct/lgeller/zeroph.html>
3. <http://www.smithsonianmag.com/history/origin-number-zero-180953392/?no-ist>
4. [http://www.livescience.com/27853-who-invented-zero.html\](http://www.livescience.com/27853-who-invented-zero.html%5C)
5. <http://yaleglobal.yale.edu/about/zero.jsp>
6. <http://www.scientificamerican.com/article/history-of-zero/>
7. <http://www.history.com/news/ask-history/who-invented-the-zero>
8. <https://www.quora.com/History-of-Mathematics/How-did-Aryabhata-discover-zero>
9. <http://www.npr.org/2015/01/03/374737120/the-zig-zagging-history-of-the-number-zero>
10. <https://www.youtube.com/watch?v=A_e5jspsNsM>
11. <http://www.mostlyodd.com/on-the-origin-of-zero/>
12. <http://gwydir.demon.co.uk/jo/numbers/interest/zero.htm>
13. [http://www.theguardian.com/notesandqueries/query/0,5753,-1358,00.html](http://www.theguardian.com/notesandqueries/query/0%2C5753%2C-1358%2C00.html)