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Latex Document

HOW TO DERIVE THE LEAST SQUARES ESTIMATES FOR $\hat{a}$ AND $\hat{b}$ GIVEN THE FOLLOWING BEGINNING POINT:

$$E = \sum_{i=1}^{n} (y_i - ax_i - b)^2$$

Part I of The Derivation

Step 1 Take the partial derivative of E with respect to b.

$$\frac{\partial E}{\partial b} = \sum_{i=1}^{n} [-2(y_i - ax_i - b)]$$

.

Step 2 Set this equation equal to zero to find the estimate for b, or $\hat{b}$.
$0 = \sum_{i=1}^{n} (-2y_i + 2\hat{a}x_i + 2\hat{b})$.

Step 3 Divide by 2 and solve for n$\hat{b}$
$n\hat{b} = \sum_{i=1}^{n} (y_i - \hat{a}x_i)$

Step 4 Divide both sides by n.
$\hat{b} = \frac{\sum_{i=1}^{n} y_i}{n} - \frac{\hat{a}\sum_{i=1}^{n} x_i}{n}$

Step 5 And simplify and equation.
$\hat{b} = \bar{y} - \hat{a}\bar{x}$

Part II of The Derivation

Step 1 Take the partial derivative of E with respect to a.

$$\frac{\partial E}{\partial a} = \sum_{i=1}^{n} [-2x_i(y_i - ax_i - b)]$$

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Step 2 Set this equation equal to zero to find the estimate for a, or $\hat{a}$.
$0 = \sum_{i=1}^{n} (-2x_i y_i + 2\hat{a}(x_i)^2 + 2\hat{b}x_i)$.

Step 3 Divide by 2 and plug in our previous statement for $\hat{b}$.
$0 = \sum_{i=1}^{n} (-x_i y_i + \hat{a}(x_i)^2 + x_i\bar{y} - \hat{a}x_i\bar{x})$.

Step 4 Simplify by gathering the $\hat{a}$.
$0 = \sum_{i=1}^{n}(\hat{a}(x_i^2 = \bar{x}x_i) - x_i y_i + x_i\bar{x})$

Step 5 Separate $\hat{a}$ from the rest of the equation and simplify.
$\sum_{i=1}^{n}(x_i y_i - x_i\bar{y}) = \hat{a}\sum_{i=1}^{n}(x_i^2 - \bar{x}x_i)$

$$\frac{\sum_{i=1}^{n}(x_i y_i) - \sum_{i=1}^{n}(x_i\bar{y})}{\sum_{i=1}^{n}(x_i^2) - \sum_{i=1}^{n}(\bar{x}x_i)} = \hat{a}$$

Step 6 Multiple both sides of the equation by one, but use $\frac{n}{n}$ as your one on the left side.

$$\frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - (\sum x_i)^2} = \hat{a}$$

In conclusion, after taking the partial derivatives with regards to a and b, we obtain the following estimates:

$$\hat{a} = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - (\sum x_i)^2}$$

$$\hat{b} = \bar{y} - \hat{a}\bar{x}$$

And after looking at these estimates as opposed to those produced by other methods, it has been determined that this, the Least Squares Method, is the best way to minimize error, thus producing a line of best fit.