

1. Prove that $\sqrt{123}$ is irrational.

Proof: if $\sqrt{123}$ is rational it can be written in the form $\frac{a}{b}$.

$$\frac{a}{b} = \sqrt{123}$$

$$b\sqrt{123} = a$$

$$123b^2 = a^2$$

Considering the last equation, we know that 123 is not a perfect square. As we have shown before the square root of any number that is not a perfect square will create a discrepancy in the prime factorizations of the two sides. Therefore $\sqrt{123}$ is irrational.

2. Find the truth table for $p \Rightarrow \neg q$.

p	q	$p \Rightarrow \neg q$
T	T	F
T	F	T
F	T	T
F	F	T

3. Negate the following statement:

$\exists w \in [a, b]$ such that $\forall x \in [a, b], f(x) \leq f(w)$.

$\forall w \in [a, b], \exists x \in [a, b], f(x) > f(w)$.

4. Let $X = \{x_1, x_2, \dots, x_n\}$. Suppose that $A \subseteq X$ and that A is represented by the binary sequence $a_1, a_2, \dots, a_n$.
a) How can the number of elements of A be determined from $a_1, a_2, \dots, a_n$?

The number of elements in A is determined by $\sum_{i=1}^n a_i$.