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Math 5010

Completing the Square

How to find zeros when Completing the Square

$$ax^2 + bx + c = 0 \tag{1}$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a} \tag{2}$$

$$x^2 + \frac{b}{a} + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2 \tag{3}$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2} \tag{4}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac}{4a^2} + \frac{b^2}{4a^2} \tag{5}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2} \tag{6}$$

$$x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} \tag{7}$$

$$x = \frac{-b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}} \tag{8}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{9}$$

What steps do we take?

Set your equation equal to zero	(1)
Divide everything by a and subtract the constant term on to the opposite side of the variable terms	(2)
Divide $\frac{b}{a}$ by two and square it then add it to both sides	(3)
Factor the left side and distribute the squared the right side	(4)
Find the common demoninator for the right side	(5)
Simplify the right side	(6)
Take the square root of both sides	(7)
Subtract $\frac{-b}{2a}$ over to the right side	(8)
And simplify	(9)

What do we end up with?

The Quadratic Equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$