

## Transcript

What is a Perfect number?

The great mathematician Euclid, who is believed to live in Alexandria, Egypt around 300 BC, was instrumental in the study of perfect numbers.

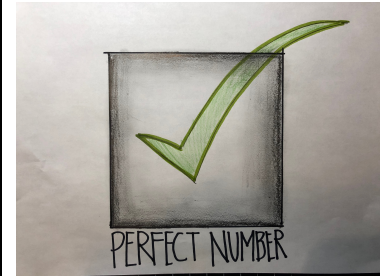
From Euclid's *Elements*, he states "If as many numbers as we please beginning from a unit are set out continuously in double proportion until the sum of all becomes prime, and if the sum multiplied into the last makes some number, then the product is perfect" (Voight).

Another words, a Perfect number is and natural where all of its proper divisors, or all of its divisors excluding itself, add up to the number itself, where the number is in the form of  $(2^n - 1)(2^{n-1})$ . So, if the number itself is included, the divisors of a perfect number would add up to two times that given number.

## Stage Directions

Cue Music:

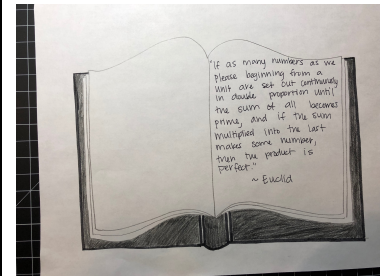
Draw:



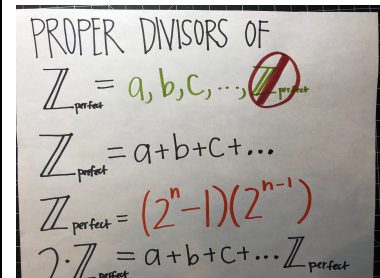
Sketch:



Draw:



Write:



The first perfect number is 6. Sixes proper divisors are 1, 2 and 3. So,  $1+2+3=6$ , thus making it a perfect number.

The next three perfect numbers are 28, 496 and 8128. 6 is the only perfect number between 1 and 10, 28, the only one between 10 and 100, 496 is the only one between 100 and 1000 and 8128 is the only perfect number between 1000 and 10000. This pattern does not hold for the subsequent perfect numbers, however, as the next one it is 8 digits long and the next 10, both ending in 6 and breaking the alternating pattern between sixes and eights.

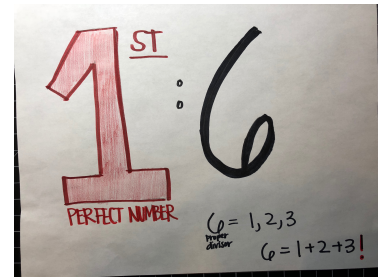
But, all perfect numbers discovered are even and also end in either a 6 or an 8.

Perfect numbers have strong biblical ties, as well, reflected in the creation of the universe.

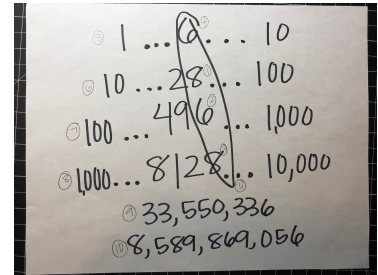
In the bible, God created the earth in 6 days and a moon orbits the Earth in 28 days.

Paul Hoffman, the author of Archimedes Revenge argued that, "it is the numbers themselves, not any connection to the empirical world, that makes them perfect."

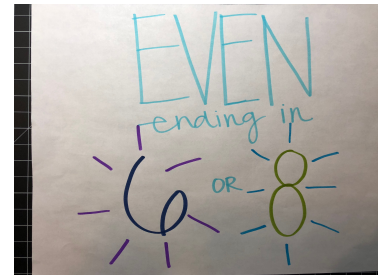
Draw:



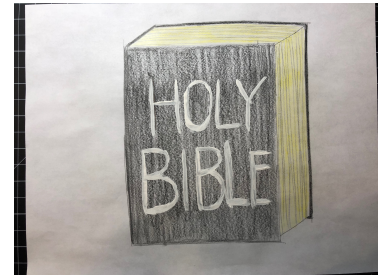
Write:



Write:



Draw:



Draw:



Saint Augustine put it as, "Six is a number perfect in itself, and not because God created all things in six days; rather the inverse is true; God created all things in six days because this number is perfect."

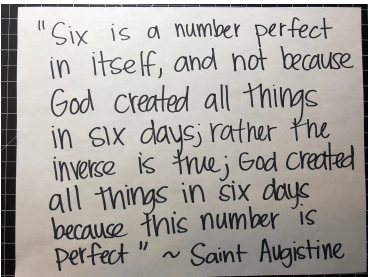
As of January 2018, there are 50 known perfect numbers, which means there are 50 known Mersenne Primes.

Wait, wait, wait? Hold up. What's a Mersenne prime?  
First, let's talk about Mersenne numbers.

A Mersenne number is a number in the form  $2^n - 1$ , where  $n$  is a natural number.

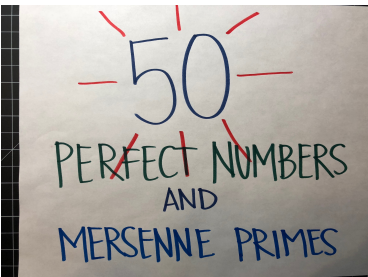
The Mersenne Primes are a subset of the Mersenne Numbers.  
So, a Mersenne Number is a Mersenne Prime if  $n$  is a prime integer and  $2^n - 1$  is also a prime integer.

Write:



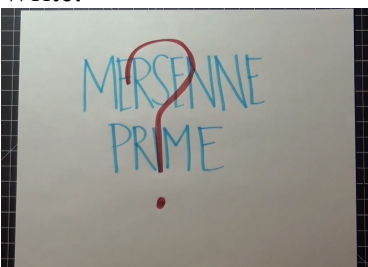
"Six is a number perfect in itself, and not because God created all things in six days; rather the inverse is true; God created all things in six days because this number is perfect" ~ Saint Augustine

Write:



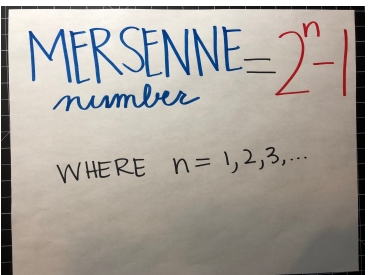
50  
PERFECT NUMBERS  
AND  
MERSENNE PRIMES

Write:



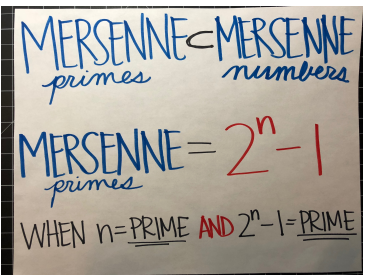
MERSENNE  
PRIME

Write:



MERSENNE =  $2^n - 1$   
number  
WHERE  $n = 1, 2, 3, \dots$

Write:



MERSENNE =  $2^n - 1$   
primes  
WHEN  $n = \text{PRIME}$  AND  $2^n - 1 = \text{PRIME}$

With every discovery of a Mersenne Prime, there is a discovery of a perfect number as well. The formula  $(2^n - 1)(2^{n-1})$  is a perfect number if, and only if  $2^n - 1$  is a Mersenne Prime. So, if  $n$  in  $2^n - 1$  is prime AND if  $(2^n - 1)$  is prime, THEN the whole equation  $(2^n - 1)(2^{n-1})$  is a perfect number.

Marin Gardner said, “One would be hard put to find a set of whole numbers with a more fascinating history and more elegant properties surrounded by greater depths of mystery—and more totally useless—than the perfect numbers” (Garcia).

Although perfect numbers have no known practical application, they are arguably the most elegant and fascinating of numbers in history.

Write:

$(2^n - 1)(2^{n-1}) = \text{PERFECT NUMBER}$   
 iff  $n = \text{PRIME NUMBER}$   
 AND  
 $(2^n - 1) = \text{PRIME NUMBER}$

Write:

“One would be hard put to find a set of whole numbers with a more fascinating history and more elegant properties surrounded by greater depths of mystery—and more totally useless—than the perfect numbers.”  
 ~ Marin Gardner

Write:

PERFECT  
 NUMBERS

End With:

Created by Nicole Westfall

References:

Garcia, A. “On Perfect Numbers.” Department of Mathematics and Computer Science. *Saint Mary’s College of California*, (17 May 2016). Retrieved from, <http://math.stmarys-ca.edu/wp-content/uploads/2017/07/Arturo-Garcia.pdf>

Hoffman, J. (1988). *Archimedes Revenge: The Joys and Perils of Mathematics*. W.W. Norton & Co., Inc. New York.

Voight, J. “Perfect Numbers: and Elementary Introduction.” *Dartmouth*, (31 May 1998).

<https://math.dartmouth.edu/~jvoight/notes/perfelem.pdf>