Albert Einstein once said, “Two things are infinite: the universe and human stupidity; and I’m not sure about the universe.” As much as I agree with Einstein here, infinity is still one of those concepts that is highly misunderstood in the world today; especially in mathematics. Have you ever heard the phrase, “to infinity, and beyond!” I mean, what is that even about? What is beyond infinity, or is there anything beyond infinity? Phillip J. Davis said, “Mathematics, in one view, is the science of infinity.” So, with all these people talking about infinity, what is it *really*? Why is it important, or is it important?

. . . interviews asking people about infinity.

As you can see, the term ‘infinity’ seems to be something everyone has heard of, but that few actually understand. First, let’s clarify some common misconceptions about infinity. Remember our Buzz Lightyear quote, “to infinity, and beyond?” This seems to hint that infinity is a destination, or that it is a number that can be reached. When in fact infinity is not a number. If you keep counting, and counting, and counting until you get tired of counting, will you be close to infinity? NO, because again, infinity is not a number! Infinity doesn’t change, infinity doesn’t ‘get bigger,’ it doesn’t grow, it just is. Infinity is the concept that helps us try to conceptualize things that don’t end, that have no bound. Let’s look at a couple examples to wrap our minds around this:

If you’ve ever taken a reasonably advanced mathematics class, you’ve probably heard about limits. Ever heard a teacher say, “the limit as *x* approaches infinity?” I definitely have, but do you see what’s wrong with that statement? x can’t ‘approach’ infinity, because infinity is not something that can be approached! Rather, we should say, “the limit as *x* becomes infinite,” implying that x is getting bigger without bound.

Have you ever heard of “the point at infinity” used by artists? In the 1500’s, artists started using a vanishing point in their paintings, giving their painting a 3-dimensional type look. The elements in the painting would be constructed such that it looked like they were disappearing towards that point. Consider, as an example, standing on an ‘infinitely’ straight road in the desert. If you look far enough, it seems like there is a point on the horizon where the road stops. That, in a painting, would be the vanishing point, or the point at infinity. Pretty cool right? However, what if you started walking down this road. Once you reach that point on the road you thought was the point at infinity, you would be looking ahead at infinitely more road, with a new point at infinity. Repeat this process over and over again, and you’ll have walked a whole lot of road and probably be really tired. And still, you will never reach this ‘point at infinity.’ Because in reality, that point really doesn’t exist! It is just a theoretical idea that artists use to paint really long roads.

So, to construct a somewhat usable definition, infinity is a term used to describe things that never end, or that have no bound.

Now, let’s turn our minds back to mathematics. To give just a little bit of history, the ancient Greeks were the first to discover infinity. And, understandably, they were a little rattled by it! Infinity has since been the subject of research for many mathematicians. One of the most notable mathematicians who studied infinity was Georg Cantor. Born in 1845 in Russia, Cantor addressed infinity from the angle of set theory. Remember all those numbers we were counting up earlier? Namely, 1,2,3,4, etc.? These numbers, called the counting numbers (aptly named, because they are the numbers we use to count), don’t end. There is no ‘largest’ counting number. So, putting all those counting numbers in to one group, we would get a set with infinitely many elements; or in other words, a countably infinite set. Cantor described the number of elements in this set (or as mathematicians like to say, the cardinality of the set) using the first letter of the Hebrew alphabet: the cardinality being aleph null. Thus, Cantor had a way of quantifying infinitely many elements using one set. Quantifying infinities in this finite manner has been extremely useful for mathematicians throughout the years.

So, if the quantity of counting numbers is aleph null, what might be the quantity of other sets of numbers; say the irrational numbers? Are there some infinities that are bigger than other infinities? Well that my friends, is a question for another time!

So next time you hear Buzz shout out, “to infinity and beyond,” think about what that means, and convince yourself that Buzz isn’t ever going to get there.